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Theoretical and experimental study of natural convection heat transfer from isothermal hemisphere

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Abstract—The simplified analytical solution and experimental study of laminar free convection heat transfer from an isothermal hemisphere in unlimited space have been presented. The solution is based on adaptation of the methods used for inclined isothermal plates. In the proposed solution the control surface of the hemisphere was considered as a small inclined surface. Inclination of this surface was not a constant one but it was a function of azimuth angle. The result of theoretical consideration is presented in the relation of Nusselt and Rayleigh numbers: $Nu = 0.533 \times Ra^{1/4}$. The comparison of theoretical solutions with experimental results presented in this paper and results of other authors shows good agreement.

INTRODUCTION

Free convective heat transfer, especially from bodies or objects limited by spherical surfaces, takes place in electronics, aeronautics, aquanauts, chemical apparatus, building engineering, lighting industry. In these branches spheres, hemispheres, bowls, copulas and capsules are very often used as insulating, constructing or shielding surfaces. However, there are many technical problems with the estimation of the coefficient of heat transfer and subsequently the convective heat losses from these spherical surfaces.

Although the studies of the literature prove that free convection from curved, especially spherical surfaces constitutes presently a classical subject of heat transfer investigations [1–9], there is very little information on free convection heat transfer from hemispheres [8–12]. The presented work attempts analytical solution of free convection in an unlimited space from a spherical surface represented by a hemisphere. Solutions of free convection flow over horizontal, vertical or inclined plates have been obtained by application of the similarity methods. However the spherical configurations do not give similarity and several other methods have been employed to obtain the solution of the governing equations. Among the earliest detailed considerations of this case are papers of Merk and Prins (1954) [13] and Acrivos (1960) [14], who assumed that the influence of natural convection is important in the thin layer of fluid adjacent to the surface (boundary layer approximation). Using this hypothesis, asymptotic solutions were calculated by these authors for the extreme case of ($Pr \Rightarrow \infty$). Chiang *et al.* (1964) [15], have employed perturbation methods, and Chen and Mucoglu (1977) [16] have used finite difference procedures, which led to numerical solutions of temperature and velocity profiles and to the local Nusselt numbers. However, their results are valid only for the front part of the sphere submerged in the air. Analytical–numerical methods of the solution of the heat transfer have been presented by Saville and Churchill (1967) [17]. Mixed theoretical and experimental methods have been employed by Stewart (1971) [18]. Raithby and Hollands (1975) [6], Raithby *et al.* (1976) [7] and Cieśliński (1985) [8] adapted for the sphere the method used by Nusselt for the laminar film condensation on a horizontal cylinder. In all of these theoretical methods of analytical solution of this problem mentioned so far, the gradient of the pressure inside the boundary layer, normal to the surface of the sphere, is neglected, so maybe this is the reason for the discrepancies between the experimental and the theoretical results of the local heat transfer coefficients on isothermal spherical surfaces indicated in ref. [10].

AIMS

This work is aimed at the development of the method of analytical solution of the simplified reduced differential equation describing the free convective heat transfer from an isothermal surface, in which the case of inclined isothermal plate [19] has been transformed into hemisphere. Proposed physical model of the phenomenon and introduced simplified assumptions are a consequence of visualisation experiments of convective heat transfer from isothermal

NOMENCLATURE

a	thermal diffusivity ($\lambda/c_p\rho$)
A	cross-section area in boundary layer, equation (15)
A_k	control surface on hemisphere, equation (16)
C_0, C_1	constants in equations (22) and (23)
C_n	coefficient in Nusselt–Rayleigh relation (31)
c_p	specific heat at constant pressure of the fluid
d	diameter
f	function (27)
F	coefficient of boundary layer shape, equations (6) and (7)
g	gravitational acceleration
h	thickness of the slot of measuring device
I	current of the heater
m	mass flux
Nu	Nusselt number ($\alpha R/\lambda$)
p	pressure
Pr	Prandtl number (ν/a)
Q	heat flux
R	radius of the hemisphere
Ra	Rayleigh number ($g\beta\Delta TR^3/(\nu a)$)
T	temperature
u	function (20)
V	voltage of the heater
W	velocity
x	tangential co-ordinate to the inclined surface
x_{loss}	coefficient of heat losses, equation (38)
y	normal co-ordinate to the inclined surface.

Greek letters

α	heat transfer coefficient
β	coefficient of volumetric expansion
γ	angular downstream location, Fig. 1
δ	thickness of boundary layer
Θ	dimensionless temperature
Δ	difference
λ	thermal conductivity
η	dynamic viscosity
ν	kinematic viscosity
ρ	fluid density.

hemisphere. The result obtained theoretically has been verified by experimental study.

RESULTS OF THE VISUALIZATION OF THE NATURAL CONVECTION FROM HEMISPHERE

Photographs presented in Fig. 1 concern the natural convective heat transfer from an isothermal hemisphere of a diameter $d = 0.06$ m. From many photos only the four representative cases of $Ra = 6.04\times10^3$, $Nu = 5.33$ (a), $Ra = 3.23\times10^4$, $Nu = 5.86$ (b), $Ra = 8.22\times10^4$, $Nu = 7.91$ (c) and $Ra = 3.54\times10^5$, $Nu = 12.5$ (d) have been chosen as an example for presentation. The tested fluid was glycerine purified by vacuum distillation. The flow pattern was detected by aluminium powder as a tracer dispersed in glycerine and illuminated by laser light-slit. Pictures were taken with a camera with exposure time equal to 8 s.

THE MODEL OF THE NATURAL CONVECTION FROM ISOTHERMAL HEMISPHERE

Several simplifying assumptions typical for the classical problem of the free convection from isothermal surfaces have been assumed such as:

fluid is incompressible and its flow is laminar and steady,

the flow is predominantly parallel to the control surface, with boundary layer unaltered with the distance along the surface ($W_y \gg W_x$),

physical properties of the fluid in the boundary layer (index x) and in the undisturbed region (∞) are constant,

thickness of thermal and hydraulic boundary layers are the same,

inertia terms, viscous dissipation and internal heat sources are neglected.

The co-ordinate system for natural convection flow adjacent to the surface of the hemisphere is presented in Fig. 2.

An infitesimal control surface of the hemisphere dA_k and of the inclined plate can be considered as the same. The only difference is that for an inclined plate the angle of inclination of the control surface $(\pi/2-\gamma)$ is constant but for a hemisphere this angle varies from $\pi/2$ to 0.

The Navier–Stokes equations for the control space inside the boundary layer extracted by differential angle of the hemisphere $d\gamma$, control surface of the hemisphere dA_k and the boundary layer thickness δ are:

$$\nu\frac{\partial^2 W_x}{\partial y^2}+g\beta(T_x-T_\infty)\sin(\pi/2-\gamma)-\frac{1}{\rho}\frac{\partial p}{\partial x}=0 \qquad (1)$$

$$g\beta(T_x-T_\infty)\cos(\pi/2-\gamma)-\frac{1}{\rho}\frac{\partial p}{\partial y}=0. \qquad (2)$$

Instead of the direct form of the Fourier–Kichhoff equation it was decided, according to Squire and

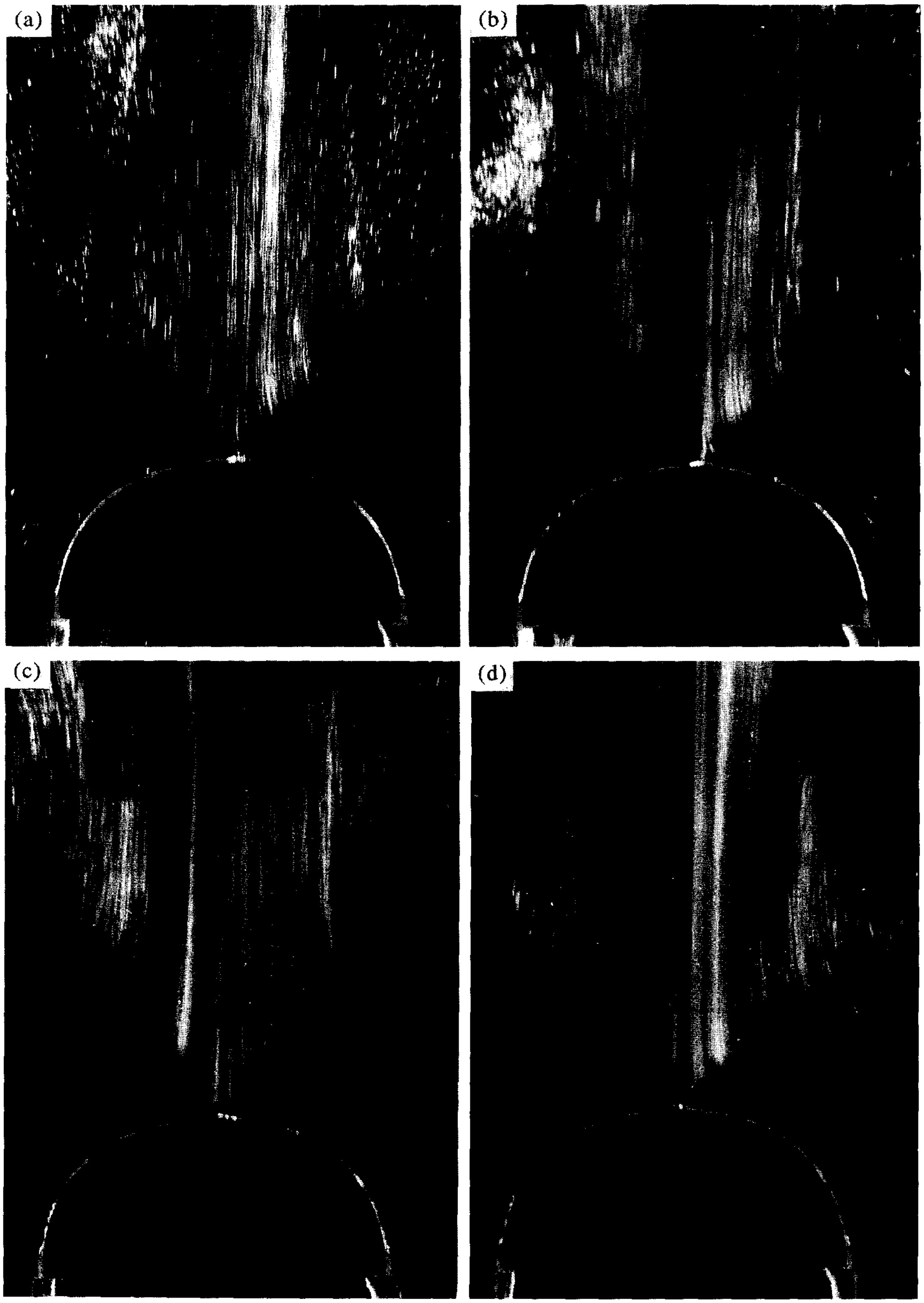

Fig. 1. Results of visual study of natural convective heat transfer from isothermal hemisphere: (a) $Ra = 6.04 \times 10^3$, $Nu = 5.33$, (b) $Ra = 3.23 \times 10^4$, $Nu = 5.86$, (c) $Ra = 8.22 \times 10^4$, $Nu = 7.91$ and (d) $Ra = 3.54 \times 10^5$, $Nu = 12.5$. These experimental points are specified by the same letters in Fig. 7, tested fluid was glycerine.

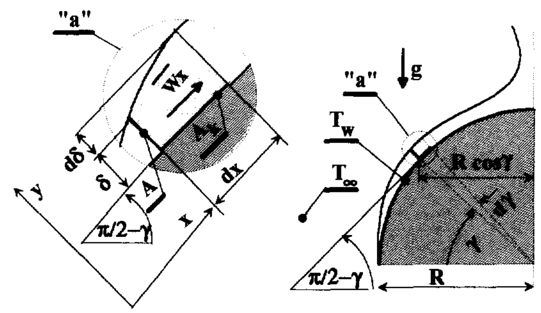

Fig. 2. Physical model of the natural convection heat transfer in an unlimited space from the isothermal hemisphere.

Eckert, to make the assumption that the temperature profile in the boundary layer is described by:

$$\theta = \frac{(T_x - T_\infty)}{(T_w - T_\infty)} = \left(1 - \frac{y}{\delta}\right)^2. \quad (3)$$

The quasi-analytical solution of equations (1)–(3), presented in ref. [19] in the form of the local and mean velocity in control space, is

$$W_x = \frac{g\beta\Delta T}{\nu}\left[\left(\frac{y^4}{12\delta^2} - \frac{2y^5}{60\delta^3} - \frac{y^2}{6} + \frac{7\delta y}{60}\right)\frac{\partial\delta}{\partial x}\sin\gamma + \left(\frac{y^2}{2} - \frac{y^3}{3\delta} - \frac{y^4}{12\delta^2} + \frac{\delta y}{4}\right)\cos\gamma\right] \quad (4)$$

$$\overline{W_x} = \frac{1}{\delta}\int_0^\delta W_x \, dy = \frac{g\cdot\beta\cdot\Delta T\cdot\delta^2}{\nu}\left(\frac{\partial\delta}{\partial x}\frac{\sin\gamma}{72} + \frac{\cos\gamma}{42}\right). \quad (5)$$

For subsequent considerations in pursuance of [19] and [20] the mean value of the boundary layer thickness $(\partial\delta/\partial x)$ increases along the angular distance from the circular leading edge was introduced and it was assumed that this value, except for the separation region ($\gamma \rightarrow \pi/2$) and at the beginning of arising of the boundary layer ($\gamma = 0$), is constant

$$\frac{\partial\delta}{\partial x} \cong \overline{\frac{\partial\delta}{\partial x}} = F = \text{idem}. \quad (6)$$

A transformation of the coefficient of the boundary layer shape on the control surface of the hemisphere from Cartesian co-ordinates into spherical ones yields:

$$F = \frac{1}{R}\overline{\frac{\partial\delta}{\partial\gamma}} = \text{idem}. \quad (7)$$

The change in mass flow intensity in control space is

$$dm = d(A \cdot \bar{W}_x \rho) \quad (8)$$

where A is the cross-sectional area of the boundary layer.

The amount of the heat necessary to create this change in mass flux is

$$dQ = \Delta i \cdot dm = \rho \cdot c_p \overline{(T_x - T_\infty)} \, d(A \cdot \bar{W}_x). \quad (9)$$

Substitution of the mean value of the temperature

$$\overline{(T_x - T_\infty)} = \frac{1}{\delta}\int_0^\delta \Delta T \cdot \left(1 - \frac{y}{\delta}\right)^2 dy = \frac{\Delta T}{3} \quad (10)$$

gives

$$dQ = \frac{\rho \cdot c_p \cdot \Delta T \cdot d(A \cdot \bar{W}_x)}{3}. \quad (11)$$

The heat flux described by equation (11) may be compared to the heat flux determined by Newton's equation (12)

$$dQ = \alpha \cdot \Delta T \cdot dA_k = -\lambda \cdot \left(\frac{\partial\theta}{\partial y}\right)_{y=0} \Delta T \cdot d/A_k \quad (12)$$

where dA_k is the control surface of the heating hemisphere.

From simplifying assumption of the temperature profile inside the boundary layer (3), the dimensionless temperature gradient on the heated surface may be evaluated as

$$\left(\frac{\partial\theta}{\partial y}\right)_{y=0} = -\frac{2}{\delta}. \quad (13)$$

Substituting equation (13) into (12) and equating the result with equation (11), one obtains dependence (14)

$$\frac{\rho \cdot c_p \delta \cdot d(A \cdot \bar{W}_x)}{6 \cdot \lambda} = dA_k. \quad (14)$$

According to Fig. 2 the cross-section area and control surface of the hemisphere are as follows

$$A = 2\pi R\delta\cos\gamma \quad (15)$$

$$dA_k = 2\pi R^2 \cos\gamma \, d\gamma. \quad (16)$$

Substitution of equations (5), (7), (15) and (16) into (14) gives

$$\frac{Ra_R}{6}\hat{\delta}\, d\left[\hat{\delta}^3 \cos\gamma\left(F\frac{\sin\gamma}{72} + \frac{\cos\gamma}{40}\right)\right] = \cos\gamma \, d\gamma \quad (17)$$

where the primes indicate the dimensionless boundary layer thickness $\hat{\delta} = \delta/R$.

Integration of equation (17) gives

$$\frac{Ra_R}{2}\hat{\delta}^3\left(F\frac{\sin\gamma}{72} + \frac{\cos\gamma}{40}\right)\frac{d\hat{\delta}}{d\gamma} - \frac{Ra_R}{6}\hat{\delta}^4\frac{\left(F\frac{\sin 2\gamma}{72} + \frac{\cos 2\gamma}{40}\right)}{\cos\gamma} = 1. \quad (18)$$

The last expression is a linear, differential and nonhomogeneous equation:

$$\frac{3}{4}\left(F\frac{\sin\gamma}{72}+\frac{\cos\gamma}{40}\right)\frac{du}{d\gamma}-u\frac{\left(F\frac{\sin 2\gamma}{72}+\frac{\cos 2\gamma}{40}\right)}{\cos\gamma}=1 \tag{19}$$

where

$$u=\frac{Ra_R}{6}\hat{\delta}^4. \tag{20}$$

The solution of the homogeneous equation (21) is the expression (22):

$$\frac{3}{4}\left(F\frac{\sin\gamma}{72}+\frac{\cos\gamma}{40}\right)\frac{du}{d\gamma}-u\frac{\left(F\frac{\sin 2\gamma}{72}+\frac{\cos 2\gamma}{40}\right)}{\cos\gamma}=0 \tag{21}$$

$$u^{-3/4}=C_0\cos\gamma\left(F\frac{\sin\gamma}{72}+\frac{\cos\gamma}{40}\right). \tag{22}$$

Allowing for the integration constant ($C_0 = C_0(\gamma)$) leads to a general solution of equation (19)

$$u=\frac{4}{3}\frac{\int_0^\gamma \cos^{4/3}\gamma\left(F\frac{\sin\gamma}{72}+\frac{\cos\gamma}{40}\right)^{1/3}d\gamma+C_1}{\cos^{4/3}\gamma\left(F\frac{\sin\gamma}{72}+\frac{\cos\gamma}{40}\right)^{4/3}}. \tag{23}$$

The integration constant (C_1) in equation (23) evaluated from the boundary condition (24) yields:

$$\text{for } \gamma=0 \quad \delta=0 \quad u=0 \quad \text{and} \quad C_1=0 \tag{24}$$

$$u=\frac{4}{3}\frac{\int_0^\gamma \cos^{4/3}\gamma\left(F\frac{\sin\gamma}{72}+\frac{\cos\gamma}{40}\right)^{1/3}d\gamma}{\cos^{4/3}\gamma\left(F\frac{\sin\gamma}{72}+\frac{\cos\gamma}{40}\right)^{4/3}}. \tag{25}$$

Elimination of variable (u) according to definition (20) gives an expression describing the shape of the boundary layer on the hemisphere

$$\hat{\delta}(\gamma)=\frac{\delta(\gamma)}{R}=\frac{2^{3/4}}{Ra_R^{1/4}}f(\gamma,F) \tag{26}$$

where

$$f(\gamma,F)=\left[\frac{\int_0^\gamma \cos^{4/3}\gamma\left(F\frac{\sin\gamma}{72}+\frac{\cos\gamma}{40}\right)^{1/3}d\gamma}{\cos^{4/3}\gamma\left(F\frac{\sin\gamma}{72}+\frac{\cos\gamma}{40}\right)^{4/3}}\right]^{1/4}. \tag{27}$$

The coefficient (F) can be estimated from its definition (equation (7))

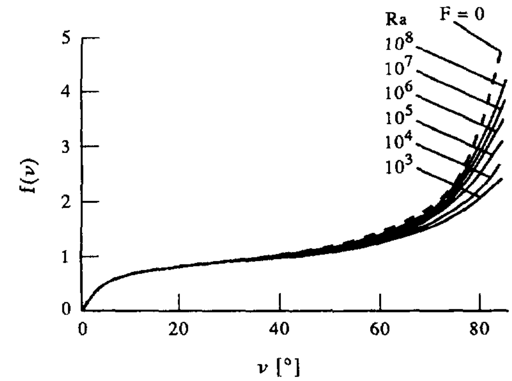

Fig. 3. Variation of $f(\gamma)$ on isothermal hemisphere.

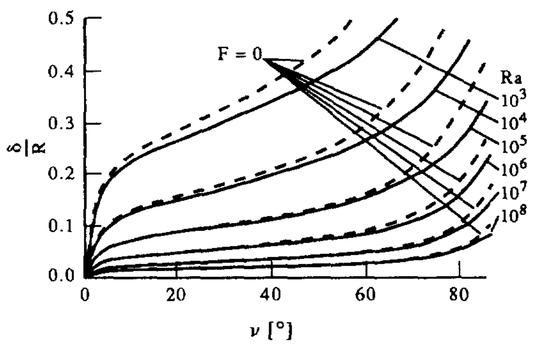

Fig. 4. Variation of nondimensional boundary layer thickness δ/R on isothermal hemisphere.

$$F=\frac{1}{R}\frac{\overline{\partial\delta}}{\partial\gamma}=\frac{1}{R\gamma}\int_0^\gamma\frac{d\delta}{d\gamma}d\gamma=\frac{1}{R\gamma}\delta|_0^\delta=\frac{\delta(\gamma)}{R\gamma}. \tag{28}$$

Taking into account equations (12), (13) and (26) the local values of the heat transfer coefficient $\alpha(\gamma)$ have the form:

$$\alpha(\gamma)=\frac{2\cdot\lambda}{\delta(\gamma)}=\frac{2^{1/4}Ra_R^{1/4}}{R\cdot f(\gamma,F)}. \tag{29}$$

The mean value of these coefficient for the entire hemisphere is:

$$\bar{\alpha}=\frac{1}{\sin\gamma}\int_0^\gamma\alpha(\gamma)\cdot\cos\gamma\cdot d\gamma. \tag{30}$$

Introduction of Nusselt number into the above equations gives the searched Nusselt–Rayleigh relation:

$$Nu_R=\frac{2^{1/4}}{\sin\gamma}\int_0^{\pi/2}\frac{\cos\gamma}{f(\gamma,F)}d\gamma Ra_R^{1/4}=C_n Ra_R^{1/4}. \tag{31}$$

The obtained result of the simplified analytical solution solves the system of four mutually correlated equations (26), (27), (28) and (31) with four unknowns $\delta(f, Ra)$, $f(F)$, $F(\delta)$ and $Ra(f)$. Simultaneous solutions of these equations for the introduced values of Rayleigh numbers ($Ra = 10^3, 10^4, 10^5, 10^6\ 10^7$ and 10^8) and angle ($\pi/2 \geq \gamma \geq 0$) had been carried out numerically [20]. The results of these calculations are presented in Figs. 3 and 4. The distributions of ($f(\gamma)$) in the function of Rayleigh number $Vs(\gamma)$ are shown in Fig. 3. Figure 4 shows a plot of the dimensionless boundary layer thickness (δ/R) vs the angular distance from the circular leading edge (γ) for introduced values of Rayleigh number ($Ra = 10^3$, 10^4, 10^5, 10^6, 10^7 and 10^8). The result of calculation for ($F = 0$) is presented in Figs. 3 and 4 by dashed lines. An analysis of the influence of the value of the boundary layer shape (F) on the correctness of obtained solution is presented below.

For the consideration of the distribution of the local boundary layer thickness or the local heat transfer coefficient on the hemisphere or for comparison of the literature local experimental results [10] this form of solution is very convenient.

In this paper the experiments were focused on overall heat transfer coefficient from the hemisphere so for verification of the method the equation (31) needs some modifications.

The introduction of equation (27) into equation (31) leads to:

$$C_n = \frac{Nu_R}{Ra_R^{1/4}} = \frac{2^{1/4}}{\sin\gamma} \times \int_0^{\pi/2} \frac{\cos^{4/3}\gamma\left(F\dfrac{\sin\gamma}{72} + \dfrac{\cos\gamma}{42}\right)^{1/3}}{\left[\displaystyle\int_0^{\pi/2} \cos^{4/3}\gamma\left(F\dfrac{\sin\gamma}{72} + \dfrac{\cos\gamma}{42}\right)^{1/3} d\gamma\right]^{1/4}} d\gamma. \tag{32}$$

The results of numerical calculations of equation (32) for $F = 0$, 0.001, 0.01, 0.1 and 1 are $C_n = 0.4154$, 0.4154, 0.4158, 0.4194 and 0.4484, respectively. From the analysis of the boundary layer theory it is obvious that the correctness of this theory grows with the decreasing of the ratio of boundary layer thickness and the linear dimension of considered surface ($\delta/R \to 0$) or ($\delta \to 0$). Decreasing of the boundary layer thickness ($\delta \to 0$) is the consequence of rising heat transfer intensity ($Ra \to \infty$). Taking into account the above consideration and the definition of the coefficient of boundary layer shape (F) (equation (28)) it is obvious that agreement of the obtained solution with boundary layer theory increases with decreasing coefficient of boundary layer shape ($F \to 0$). For the next considerations we have assumed the lower boundary values of the coefficient in Nusselt–Rayleigh.

Therefore the free laminar convective heat transfer from an isothermal hemisphere may be expressed as the following relations between Nusselt and Rayleigh numbers:

$$Nu_R = 0.415 \quad Ra_R^{1/4} \quad \text{or} \quad Nu_d = 0.494 \quad Ra_d^{1/4}. \tag{33}$$

VERIFICATION OF THE ANALYTICAL SOLUTION

The solution of the local heat transfer coefficient equation (29), may be expressed as follows:

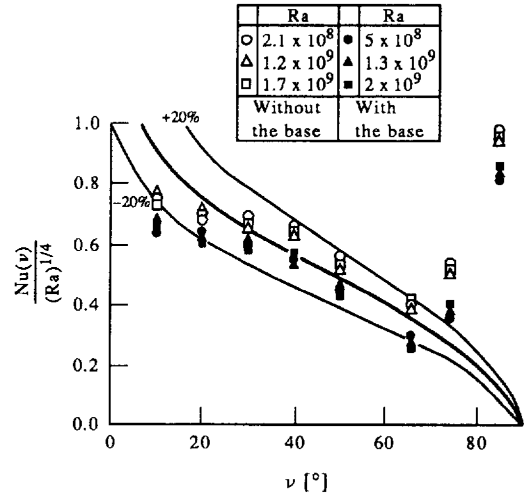

Fig. 5. The analytical solution of local heat transfer coefficient from isothermal hemisphere compared with experimental results of Jaluria and Gebhart [10]. Dark points are concerned with hemisphere with base, white points were obtained for hemisphere without the base.

$$C = Nu_R/Ra_R^{1/4} = 2^{1/4}/f(\gamma, F). \tag{34}$$

Equation (34) and experimental results of Jaluria and Gebhart [10] were compared in Fig. 5. These results agree quite well. The differences between the Jaluria's experiments and the solution for the angle ($\gamma > 75°$) is the effect of turbulization in free buoyant stream above the hemisphere. This turbulization is a factor of intensification in heat transfer, which has not been taken into account in the theoretical model of the process.

Also the agreement of obtained solution of the overall convective heat transfer from the hemisphere with the data of other authors is fairly good. For instance, the difference between the presented solution (33) and Stewart and Johnson's experimental result of the laminar free convection heat transfer from an isothermal hemisphere characterised by an average Nusselt–Rayleigh correlation $Nu_R = 0.490 Ra_R^{1/4}$ for $2.8 \times 10^5 < Ra_R < 2.8 \times 10^7$, $Pr = 0.7$ equals only 9.4% [11]. However, for Jaluria and Gebhart's data obtained for $2.5\ 10^7 < Ra < 2.5 \times 10^8$ and recalculated in [11] as $Nu_R = 0.530 Ra_R^{1/4}$ this difference is almost twice as big (18.3%). For the relationship published by Cieśliński and Pudlik, obtained for $1.2 \times 10^6 < Ra_{R,\gamma} < 1.9 \times 10^7$ and glycerine [9] and recalculated for a hemisphere which gives $Nu_{R,\gamma} = 0.548 Ra_{R,\gamma}^{1/4}$ the difference is also bigger (29.9%), but in both cases the tested hemisphere was tested with the base.

EXPERIMENTAL APPARATUS

Figure 6 is a schematic cross-section of the apparatus used. The experimental apparatus was a Plexiglas tank. The main dimensions of the tank were 0.5 m in diameter and 0.4 m in height. In the centre of the

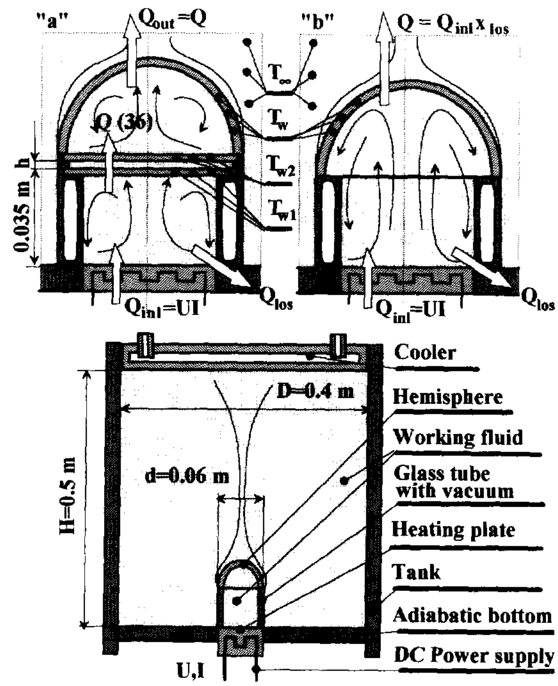

Fig. 6. The experimental set-up, (a) version with direct method of measuring heat transfer from hemisphere, (b) version with indirect method of the heat stream estimation.

bottom, on a round horizontal heating plate, was a glass tube held a round bakelite device with the copper hemisphere [Fig. 6(a)]. The tube had double walls with vacuum between them to minimize heat losses. The device consisted of two parallel round copper plates.

The heat flux from the heating plate inside the glass tube, filled by test fluid, was transported by turbulent convection into the hemisphere. This flux was transported through the slot in the measuring device by conduction. The test fluid was also inside the slot and inside the hemisphere. Heat flux from the surface of the hemisphere to surrounding test fluid was transferred again by convection, but in this case by the laminar one.

Nine thermocouples were used to measure the surface temperature; three for the hemisphere (T_w), three for the lower (T_{w1}) and three for the upper (T_{w2}) plate of the measuring devices.

Each thermocouple was soldered into holes of copper surface with the tips of about 0.001 m.

Four thermocouples were used to measure the bulk temperature (T_∞) of the fluid (dehydrated glycerine, technical glycerine and distilled water) at different levels in the tank. The inaccuracy of the temperature measurement did not exceed ± 0.1 K. Establishing of different steady states was done by a cooling system located at the top of the tank. This system consisted of a copper coil connected to a thermostat. During the experimental runs the surface temperatures of the hemisphere, the lower and the upper plates of the device, bulk temperature of the fluid and the voltage (U) and current of the heater inside the heating plate (I) were measured. All these data were recorded during established steady states.

The experiments were carried out in a hermetically closed vessel using dehydrated glycerine, technical glycerine and distilled water as the test fluids. The density, thermal expansion coefficient and the dynamic viscosity of the glycerine were experimentally determined after each experimental run. The thermal conductivity of the glycerine and other physical properties for water were taken from the published data.

Two methods of experimental procedure: first with the direct way of estimated heat stream from the hemisphere to the fluid tested and second one with indirect procedure of determining this stream were used.

Method I

In this method the heat flux transferred from electrically heated plate through the glass cylinder into the hemisphere was measured by means of the measuring device. The heat stream inflowing into the hemisphere and the heat flux from the hemisphere to the fluid was the same. In order to estimate this flux the distance between the copper round plate of measuring device (h) was estimated from equation:

$$g\beta(T_{w1}-T_{w2})h^3/(\nu a) < Ra_{cr} = 1700. \tag{35}$$

The distance of (h = 1.9 mm) was used in our experiments. Under these circumstances the convective part of heat flux inside the slot was negligibly small and the conductive heat flux was calculated according to formula:

$$Q = (\lambda/h)(T_{w1}-T_{w2})(\pi d^2/4) \tag{36}$$

where (γ) is the thermal conductivity of the liquid inside the slot at a characteristic temperature ($T_{ch} = (T_{w1}-T_{w2})/2$). The slot was filled by the tested fluid.

In this method of measuring of heat transfer stream the time of reaching the thermal balance was very long (> 5 h for any experimental points), so we decided to modify this method.

Method II

The only difference, for method II, was that the hemisphere was mounted not on a bakelite device [Fig. 6(a)] but direct on the glass tube [Fig. 6(b)]. From previous experimental results expressed by Nusselt–Rayleigh relation one can calculate and compare the heat stream from the hemisphere (Q_{out}) being the stream (Q) of the heat transferred through the slot (36) with the stream of the heat from the electrically heated plate in the bottom of the tank (Q_{in} = UI). This comparison allowed us to estimate the stream of overall heat losses (Q_{loss}) and coefficient of heat losses (x_{loss}) in the function of Rayleigh number. In this indirect method the heat stream from hemisphere was calculated from relation:

$$Q = U \cdot I \cdot x_{loss} \tag{37}$$

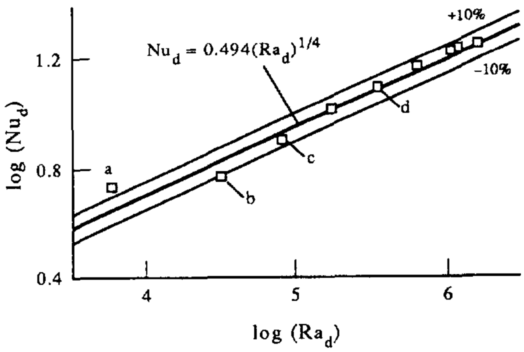

Fig. 7. The experimental results (points) of natural convective heat transfer from isothermal hemisphere of the diameter $d = 0.06$ m obtained for dehydrated glycerine with the use of method I. The analytical solution is expressed by the solid lines. The experimental points described by letters are presented in Fig. 1 in the form of photos.

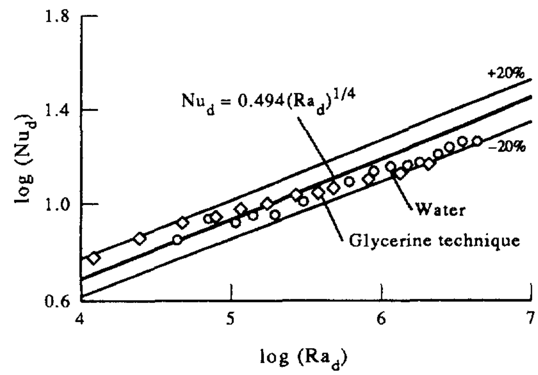

Fig. 8. The experimental results of natural convective heat transfer from an isothermal hemisphere of diameter $d = 0.06$ m obtained for technical glycerine (diamonds) and distilled water (round points) with the use of method II. The analytical solution is expressed by the solid line.

where the coefficient of heat losses evaluated from results of the experiments performed with the use of direct form of measuring of heat stream was calculated from empirical relation:

$$x_{\text{loss}} = 0.0547 \times Ra^{0.09595}. \tag{38}$$

The time taken to obtain a thermal equilibrium and perform the experiment was only about 2 h for one experimental point.

Comparison showed the results obtained with the use of these two methods to differ from each other by only 1.8%.

EXPERIMENTAL RESULTS

With dehydrated glycerine as the working fluid a series of experimental runs according to method I was made. Using the least square method the experimental results obtained for various heat streams may be correlated by Nusselt–Rayleigh relations:

$$Nu_d = 0.510 \times Ra_d^{1/4} \tag{39}$$

for: $6.04 \times 10^3 < Ra_d < 1.63 \times 10^6$ and $Pr \approx 1200$.

The results obtained using method II to estimate the heat transfer stream were correlated:

$$Nu_d = 0.501 \times Ra_d^{1/4} \tag{40}$$

for: $1.2 \times 10^4 < Ra_d < 2.0 \times 10^6$ and technical glycerine as tested fluid ($Pr \approx 700$) and

$$Nu_d = 0.444 \times Ra_d^{1/4} \tag{41}$$

for: $4.4 \times 10^4 < Ra_d < 4.1 \times 10^6$ and distilled water ($Pr \approx 6$).

The experimental results presented in the form of average Nusselt numbers (Nu_d) vs Rayleigh numbers (Ra_d) are shown and compared with analytical solution in Figs. 7 (method II) and 8 (method II).

Comparison of the experimental results with analytical solution equation (33) obtained for natural convective heat transfer from an isothermal hemisphere demonstrates that all the results fall within a -10.1% (water method II) to $+3.2\%$ (glycerine method I) range, hence it may be stated that verification of the solution suggested in this paper is correct.

CONCLUSIONS

The natural convection heat transfer in unlimited space from isothermal hemisphere has been experimentally and theoretically investigated. The analytical solutions are in good agreement with theoretical and experimental results of other authors and with experimental results presented in this paper.

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